奧冠教育中心

OLYMPIAD CHAMPION EDUCATION CENTRE

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香港國際數學競賽初賽 2019（香港賽區）

HONG KONG INTERNATIONAL MATHEMATICAL OLYMPIAD HEAT ROUND 2019 (HONG KONG REGION)

高中組 Senior Secondary Group

時限：60 分鐘

Time allowed: 60 minutes

試題

Question Paper

考生須知：

Instructions to Contestants:

1. 本卷包括 試題 乙份，試題紙不可取走。
   Each contestant should have ONE Question-Answer Book which CANNOT be taken away.
2. 本卷共 5 個範疇，每範疇有 4 題，共 20 題，每題 3 分，總分 60 分，答錯不扣分。
   There are 5 exam areas and 4 questions in each exam area. There are a total of 20 questions in this Question-Answer Book. Each carries 3 marks. Total score is 60 marks. No points are deducted for incorrect answers.
3. 請將答案寫在 答題紙 上。
   All answers should be written on ANSWER SHEET.
4. 比賽期間，不得使用計算工具。
   NO calculators can be used during the contest.
5. 本卷中所有圖形不一定依比例繪成。
   All figures in the paper are not necessarily drawn to scale.
6. 比賽完畢時，本試題會被收回。
   This Question-Answer Book will be collected at the end of the contest.

請以最簡形式填寫答案，若計算結果是分數，請確保為真分數或帶分數，或將計算結果寫成小數。錯誤單位將不給予任何分數。

Write down the answer in the simplest form. If the calculation result is a fraction, please write down the answer as a proper or mixed fraction, decimal figure is also accepted. Marks will NOT be given for incorrect unit.

請將答案寫在 答題紙 上。
All answers should be written on the ANSWER SHEET.

本試題不可取走。
THIS Question-Answer Book CANNOT BE TAKEN AWAY.
未得監考官同意，切勿翻閱試題，否則參賽者將有可能被取消資格。
DO NOT turn over this Question-Answer Book without approval of the examiner.
Otherwise, contestant may be DISQUALIFIED.

請以最簡形式填寫答案，若計算結果是分數，請確保為真分數或帶分數，或將計算結果寫成小數。錯誤單位將不給予任何分數。
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填空題（第 1 至 20 題）（每題 3 分，答錯及空題不扣分）

Open-Ended Questions (1^{st} ~20^{th}) (3 points for correct answer, no penalty point for wrong answer)

Logical Thinking
邏輯思維

1. Given *A*, *B* and *C* are three non-zero digits and the 3-digit numbers formed by these three digits have the following properties:
   1. $\overline{CBB}$ is divisible by 12;
   2. $\overline{CAC}$ is divisible by 11;
   3. $C > B > A$.

   Find the 3-digit number $\overline{ABC}$.

   已知 *A*、*B* 和 *C* 為三個非零數位且利用這三個數位組成的三位數有以下性質：
   1. $\overline{CBB}$能被 12 整除；
   2. $\overline{CAC}$ 能被 11 整除；
   3. $C > B > A$

   求三位數 $\overline{ABC}$。

2. There are *n* lines that are not parallel with each other on a plane. There are no 3 lines intersecting at a point. If they intersect 253 times, find *n*.

   平面上有 *n* 條互不平行的線，且任何三線並不共點。它們共有 253 個交點，求 *n*。

3. Alice goes southwest for 29km, then goes southeast for 30km, goes southeast for 5km and goes northwest for 37km. How far is she now from the original position?

   愛麗絲向西南走了 29 公里，向東南走了 30 公里，向東北走了 5 公里，向西北走了 37 公里，問她和原來位置相距多遠？

4. There are 15 problems in a competition. The scores of each problem are allocated in the following ways: 2 marks will be given for a correct answer, 1 marks will be deducted from a wrong answer and 0 marks will be given for a blank answer. Find the minimum number of candidate(s) to ensure that 2 candidates will have the same scores in the competition.

   某比賽共有 15 條題目。以下述方式為每個題目評分：答對得 2 分、答錯倒扣 1 分、不作答得零分。求最小參賽者的數目才可保證比賽中有二人同分。

請將答案寫在 答題紙 上。
All answers should be written on the ANSWER SHEET.

Algebra
代數

5. Find the last-digit of $7C_1^7 + 7^2C_2^7 + 7^3C_3^7 + ... + 7^6C_6^7 + 7^7C_7^7$.
求 $7C_1^7 + 7^2C_2^7 + 7^3C_3^7 + ... + 7^6C_6^7 + 7^7C_7^7$ 的個位值。

6. If $x^3 + \frac{1}{x^3} = 198$, find the value of $x + \frac{1}{x}$.
若 $x^3 + \frac{1}{x^3} = 198$，求 $x + \frac{1}{x}$ 的值。

7. If $3x = \cfrac{1}{2 - \cfrac{1}{2 - \cfrac{1}{2 - ...}}}$, find the value of x.
若 $3x = \cfrac{1}{2 - \cfrac{1}{2 - \cfrac{1}{2 - ...}}}$，求 x 的值。

8. If k is an integer, find the smallest value of k such that $(k+1)x^2 + (k-2)x - 4 = 0$ has no real roots.
若 k 為整數，求 k 的最小值使得 $(k+1)x^2 + (k-2)x - 4 = 0$ 沒有實根。

Number Theory
數論

9. Find the smallest positive integral solution of congruence equations $\begin{cases} 4x \equiv 1 (\text{mod } 7) \\ 3x \equiv 6 (\text{mod } 9) \end{cases}$
求同餘方程組 $\begin{cases} 4x \equiv 1 (\text{mod } 7) \\ 3x \equiv 6 (\text{mod } 9) \end{cases}$ 的最小正整數解。

10. Now is December. Which month will it be after 3^{81} months?
現在是十二月，3^{81} 個月後是幾月?

11. How many positive integer(s) x is / are there so that $\sqrt{x^2 - 26x + 249}$ is an integer?
有多少個正整數 x 使 $\sqrt{x^2 - 26x + 249}$ 為整數?

請以最簡形式填寫答案，若計算結果是分數，請確保為真分數或帶分數，或將計算結果寫成小數。錯誤單位將不給予任何分數。
Write down the answer in the simplest form. If the calculation result is a fraction, please write down the answer as a proper or mixed fraction, decimal figure is also accepted. Marks will NOT be given for incorrect unit.

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All answers should be written on the ANSWER SHEET.

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12. If $\frac{2019!}{12^n}$ is a positive integer, find the maximum integral value of n.

若$\frac{2019!}{12^n}$是個正整數，求 n 的最大整數值。

請以最簡形式填寫答案，若計算結果是分數，請確保為真分數或帶分數，或將計算結果寫成小數。錯誤單位將不給予任何分數。
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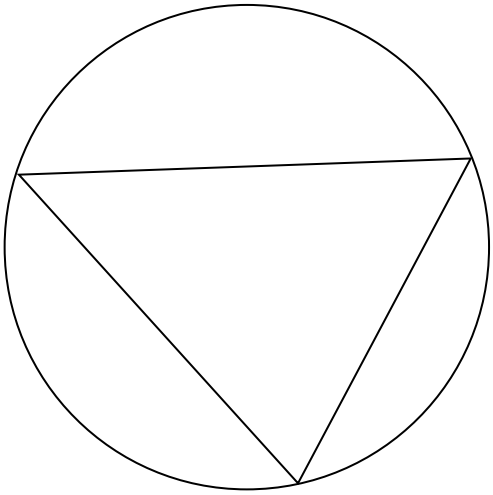
Geometry
幾何

13. Find the area enclosed by the y-axis and the straight lines $x = 3y - 6$ and $3y = 21 - 4x$.
求由 y-軸及直線 $x = 3y - 6$ 及 $3y = 21 - 4x$ 圍出的面積。

14. A circle in the figure below has radius 2. The ratio of angle in triangle is $3:4:5$. Find the area of triangle. (Answer in surd form)
下圖中的圓的半徑為 2，三角形內角比例為 $3:4:5$。求三角形的面積。(答案以根式表示)



Question 14
第 14 題

15. Find the area of regular octagon which diagonal length is 4. (Answer in surd form)
求對角線長 4 的正八邊形的面積。(答案以根式表示)

16. Given that $\tan x = \frac{\sqrt{5}}{2}$. Find the value of $\sin 2x$.
已知 $\tan x = \frac{\sqrt{5}}{2}$，求 $\sin 2x$ 的值。

請以最簡形式填寫答案，若計算結果是分數，請確保為真分數或帶分數，或將計算結果寫成小數。錯誤單位將不給予任何分數。
Write down the answer in the simplest form. If the calculation result is a fraction, please write down the answer as a proper or mixed fraction, decimal figure is also accepted. Marks will NOT be given for incorrect unit.

請將答案寫在 答題紙 上。
All answers should be written on the ANSWER SHEET.

Combinatorics
組合數學

17. There are 3 boys and 3 girls. If we need to split them into 2 rows, how many way(s) is/are there to ensure at least 2 girls in a row?
有 3 名男孩和 3 名女孩，若我們把他們分成 2 行，有多少種方法能令至少 2 名女孩在同一行?

18. In how many possible ways can 10 identical flowers be distributed to 4 distinct vases that the number of vase(s) with flower must be not less than the number of vase(s) without flower?
把 10 支完全相同的花放入 4 個不同的花瓶中，有花的花瓶不能比沒花的少，問共有多少個不同的分配方法?

19. A fair 6-face dice is thrown 3 times. Find the probability that the sum of numbers is 2-digit number.
擲一枚均質六面骰子三次。求擲得點數總和為雙位數的概率。

20. There are 9 identical blue boxes, 2 identical red boxes and 1 identical white boxes are put from left to right. How many way(s) of arrangement is / are there?
由左至右放置 9 個相同的藍色箱子，2 個相同的紅色箱子和 1 個相同的白色箱子，有多少個不同排列方法?

~ 全卷完 ~
~ End of Paper ~